

Singular value maximization in $(0, 1)$ -matrices

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Abstract

We concentrate on the problem of maximizing the largest singular value of a $(0, 1)$ -matrix, with a given number of ones and given sum of dimensions. We conjecture that if the $(0, 1)$ -matrix A is a maximizing the largest singular value it is rank one or two, and we present a number of situations in which our conjecture is correct. The results provide various conditions on A .

The key role plays here a normalized, left justified matrix, a single additional row matrix (or SARM) and a bled matrix. First we show, that a normalized, left justified matrix is a matrix maximizing the largest singular value, if we cannot construct a rank one matrix. Then we prove that a SARM is maximal among rank two matrices and we determine the maximizer among SARM's. We show, that a bled matrix B gives us the lower bound for the largest singular of the given normalized, left justified $(0, 1)$ -matrix. We also improve that bound by using normalized eigenvector of BB^T . Next we use singular value inequalities involving bled matrices to prove several results in which if A is a matrix of rank greater than two, there is a SARM with a greater largest singular value.

Keywords

Singular value, Single additional row matrix, Bled matrix.