Commuting pairs of patterns and symmetric realizations

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Abstract

The patterns that commute with tridiagonal and with other tree patterns are studied and formulae are given that relate the entries of commuting instances. For any pattern that commutes with an irreducible tridiagonal pattern, the commutativity can be realized by a complex symmetric pair. This is not so for real symmetric matrices, and an 8-by-8 example is given, resolving a natural question but raising several others.