Defect of a Kronecker product of Fourier matrices

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Abstract

Consider a real space of directions, moving into which from U, an $N \times N$ unitary matrix with no zero entries, we do not disturb the Gram matrix U^*U as well as the doubly stochastic matrix $B_{ij} = |U_{ij}|^2$ in the first order. The dimension of this space, diminished by $2N-1$ being the dimension of $\{D_r U D_c \ : \ D_r, D_c \ \text{ unitary diagonal}\}$, is called the defect of U: $\mathbf{d}(U)$.

We present different characterizations of $\mathbf{d}(F)$, where $F = F_{N_1} \otimes \ldots \otimes F_{N_r}$, of the total size N, is a Kronecker product of Fourier matrices $[F_N]_{ij} = e^{\mathbf{i}\frac{2\pi}{N}ij}$ of different size. These characterizations are related to the group $Z_{N_1} \times \ldots \times Z_{N_r}$ and the group of its one dimensional reprezentations (i.e. the group of rows of F under entrywise product).

We provide formulas expressing $\mathbf{d}(F)$ and show the multiplicativity of the quantity $\mathbf{d}(F) + (2N-1)$ with respect to Kronecker subproducts of F chosen in a special way.

We indicate applications of $\mathbf{d}(F)$ in the study of complex Hadamard matrices, as well as the problem of orthostochasticity.

Keywords

Fourier matrix, Kronecker product, Complex Hadamard matrix, Critical point.