Optimal Gersgorin-style estimation of the largest singular value

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Abstract

Let $M_n(\mathcal{C})$ be the set of all n-by-n complex matrices. For a given matrix $A = (a_{ij}) \in M_n(\mathcal{C})$, we set $P_k(A) = \sum_{j \neq k} \mid a_{k,j} \mid$, $k = 1, \ldots, n$, and define the class $\Lambda(A)$ of matrices equiradial with A by $\Lambda(A) = \{B = (b_{i,j}) \in M_n(\mathcal{C}) : |D(B)| = |D(A)| \text{ and } P_k(B) = P_k(A), k = 1, \ldots, n\}$, where, for an $X = (x_{i,j}) \in M_n(\mathcal{C})$, $D(X) = diag(x_{1,1}, \ldots, x_{n,n})$. Moreover by $\sigma_1(A)$ we denote the largest singular value of A.

In [1] it was studied the question "'what is $\max_{X \in \Lambda(A)} \ \sigma_1(X)$" and it was proved that its solution is one of the nonnegative matrices $A^{(s,k)} = (a_{i,j}^{(s,k)})$, where $s, k \in \{1, \ldots, n\}$ and $s \neq k$, such that

$$a_{ij}^{(s,k)} = \begin{cases} |a_{i,i}| & \text{for } i = j, \\ P_i(A) & \text{for } i \neq j \text{ and } j = s, \\ P_s(A) & \text{for } (i,j) = (s,k), \\ 0 & \text{otherwise.} \end{cases}$$

In this talk we investigate properties of matrices $A^{(s,k)}$ and use them to identify the solution of the mentioned question.

Keywords

Spectral norm, Largest singular value, Gersgorin data.

References

[1] Johnson, C.R., T. Szulc, and D. Wojtera-Tyrakowska (2005). Optimal Gersgorin-style estimation of extremal singular values. *Linear Algebra Appl. 402*, 46–60.