Hyponormality and Aluthge transformation of indefinite type

Rute Lemos[1], **Natália Bebiano**[2], **João da Providência**[2] **and Graça Soares**[3]

[1] *University of Aveiro, Portugal*
[2] *University of Coimbra, Portugal*
[3] *University of Trás-os-Montes e Alto Douro, Vila Real, Portugal*

Abstract

Let $\mathcal{H}$ be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and T an operator defined on $\mathcal{H}$. The Aluthge transformation of T was introduced in 1990 in a paper concerning p-hyponormal operators [1] and it has been extensively studied since then. Let J be a selfadjoint involution on $\mathcal{H}$ and let us consider $\mathcal{H}$ with a Krein space structure where the indefinite inner product is given by $[x, y] = \langle Jx, y \rangle$ for all $x, y \in \mathcal{H}$. We present an indefinite complete form of Furuta inequality inspired by [5]. We introduce the notion of J-generalized Aluthge transformation for operators T with J-polar decomposition $T = U|T|_J$, where U is J-unitary and $|T|_J$ is the J-modulus of T, and study some of its properties concerning p-hyponormal operators on Krein spaces. Some indefinite versions of well known results [1], [2], [3], [4] are obtained.

Keywords

Generalized Aluthge transformation, J-polar decomposition, J-contraction, J-hyponormal operator, Indefinite complete form of Furuta inequality.

References

[1] Aluthge, A. (1990). On p-hyponormal operators for $0 < p < 1$. *Integral Equations Operator Theory 13*, 307–315.

[2] Furuta, T. and M. Yanagida (1997). Further extensions of Aluthge transformation on p-hyponormal operators. *Integral Equations Operator Theory 29*, 122–125.

[3] Huruya, T. (1997). A note on p-hyponormal operators. *Proc. Amer. Math. Soc. 125*, 3617–3624.

[4] Yoshino, T. (1997). The p-hyponormality of the Aluthge transformation. *Interdiscip. Inform. Sci. 3*, 91–93.

[5] Yuan, J. and Z. Gao (2008). Complete form of Furuta inequality. *Proc. Amer. Math. Soc. 136*, 2859–2867.