Statistical distance measures and the Fisher information matrix

André Klein[1], **Guy Mélard**[2], **and Peter Spreij**[1]

[1] *University of Amsterdam, The Netherlands*
[2] *Université Libre de Bruxelles, Belgium*

Abstract

The purpose of this talk consists of displaying some results on statistical distance measures and its interaction with some structured matrices. In a statistical context the straight-line or Euclidean distance is unsatisfactory. This is because the coordinates or variables represent measurements that are subject to random fluctuations of different magnitudes. It is therefore important to consider a distance that takes the variability of these variables into account when determining its distance from a fix point. In the present presentation we consider the statistical distance for the case the x_i measurements do not vary independently of the x_j measurements. A rotation of the n-dimensional coordinate system through an angle ψ is considered while keeping the scatter of points given by the data fixed and label the rotated axes $\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_n$. The statistical distance measure obtained is determined entirely by the size of statistical fluctuations through the covariances. It is then identified with an appropriate quadratic form.
We shall interconnect the Fisher information matrix to the statistical distance measures. More recently, in the quantum information theory literature, an interconnection between a statistical distance measure and the Fisher information is established at the scalar level, see [3], [2], and [1]. However, we shall involve the Sylvester resultant matrix and emphasize the role structured matrices play in generalizing and better understanding the distance measures under study.

Keywords

Statistical distance measure, Rotation matrix, Fisher information matrix, Sylvester resultant matrix.

References

[1] Barndorff-Nielsen, O.E. and R.D. Gill (2000). Fisher information in quantum statistics. *J. Phys. A 30*, 4481–4490.

[2] Braunstein, S.L. and C.M. Caves (1994). Statistical distance and the geometry of quantum states. *Phys. Rev. Lett. 72*, 3439–3443.

[3] Kok, P. (2006). Tutorial: Statistical distance and Fisher information.