Why are those exponents so critical?

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Abstract

Let $\mathcal{C}$ be a class of matrices in which a continuous powering $A^{(t)}$ is defined. By a "critical exponent" for $\mathcal{C}$ we mean the smallest real number e such that for all $t > e$ and all A in $\mathcal{C}$, $A^{(t)}$ lies in $\mathcal{C}$. Of course, many classes are not closed under continuous exponentiation but may still have a critical exponent; some classes may not have a critical exponent, and it may well be possible to know that a class has a critical exponent, without knowing the exact value. We survey both historical and recent results about critical exponents, including ones for such classes as doubly nonnegative, totally nonnegative, M-matrices, inverse M-matrices, etc. and such powering as conventional, and Hadamard, etc.