The role of Kemeny's constant in properties of Markov chains

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Abstract

In a finite m-state irreducible Markov chain with stationary probabilities $\{\pi_i\}$ and mean first passage times m_{ij} (mean recurrence time when $i = j$) it was first shown, by [5], that $\sum_{j=1}^{m} \pi_j m_{ij}$ is a constant, K, not depending on i. This constant has since become known as Kemeny's constant ([2]). We consider a variety of techniques for finding expressions for K, derive some bounds for K, and explore various applications and interpretations of this result. Interpretations include the expected number of links that a surfer on the World Wide Web, located on a random page needs to follow before reaching a desired location ([7]), as well as the expected time to mixing in a Markov chain ([3]). Various applications have been considered including some perturbation results ([3], [1]), mixing on directed graphs ([6]), and its relation to the Kirchoff index of regular graphs ([8], [9]). In some further extensions it is shown that typically the variances of the mixing times depend on i ([4]).

Keywords

Markov chains, Stationary distributions, Mean first passage times, Kemenys constant, Mixing times, Perturbations, Regular graphs.

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