On the eigenvalues of principal submatrices of $J$-normal matrices: the $J$-diagonalizable case and the general $3 \times 3$ case

Natália Bebiano$^1$, Susana Furtado$^2$
and João da Providência$^1$

$^1$University of Coimbra, Portugal
$^2$University of Porto, Poland

Abstract

Let $M_n$ be the algebra of $n \times n$ complex matrices and let $J$ be a diagonal involution. Consider the indefinite inner product $\langle \cdot, \cdot \rangle$ defined by $\langle x, y \rangle = y^* J x$, $x, y \in \mathbb{C}^n$. A matrix $A \in M_n$ is said to be $J$-normal if $A^# A = AA^#$, in which $A^#$ is the $J$-adjoint of $A$ defined by $\langle Ax, y \rangle = \langle x, A^# y \rangle$ for any $x, y \in \mathbb{C}^n$ (that is, $A^# = J A^* J$). We say that $U \in M_n$ is a $J$-unitary matrix if $U^{-1} = U^#$. A matrix $A$ is $J$-unitarily diagonalizable if there exists a $J$-unitary matrix $U$ such that $U^# AU$ is diagonal.

In this talk we consider the following problem: give necessary and sufficient conditions for the existence of a $J$-normal matrix $A$ with prescribed eigenvalues for $A$ and some of its $(n - 1) \times (n - 1)$ principal submatrices. The particular case in which $A$ is $J$-unitarily diagonalizable is considered. The general $3 \times 3$ case is also studied.