# On the eigenvalues of principal submatrices of $J$-normal matrices: the J-diagonalizable case and the general $3 \times 3$ case

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**Abstract**

Let $M_n$ be the algebra of $n \times n$ complex matrices and let $J$ be a diagonal involution. Consider the indefinite inner product $[\cdot, \cdot]$ defined by $[x, y] = y^* Jx$, $x, y \in \mathbb{C}^n$. A matrix $A \in M_n$ is said to be $J$-normal if $A^\# A = AA^\#$, in which $A^\#$ is the $J$-adjoint of $A$ defined by $[Ax, y] = [x, A^\# y]$ for any $x, y \in \mathbb{C}^n$ (that is, $A^\# = JA^* J$). We say that $U \in M_n$ is a $J$-unitary matrix if $U^{-1} = U^\#$. A matrix $A$ is $J$-unitarily diagonalizable if there exists a $J$-unitary matrix $U$ such that $U^\# AU$ is diagonal.

In this talk we consider the following problem: give necessary and sufficient conditions for the existence of a $J$-normal matrix $A$ with prescribed eigenvalues for $A$ and some of its $(n-1) \times (n-1)$ principal submatrices. The particular case in which $A$ is $J$-unitarily diagonalizable is considered. The general $3 \times 3$ case is also studied.