Positive semidefinite polynomials and sums of squares

Carla Fidalgo[1] **and Alexander Kovačec**[2]

[1] *Coimbra Institute of Engineering, Portugal*
[2] *University of Coimbra, Portugal*

Abstract

By a *diagonal minus tail* form (of even degree) we understand a real homogeneous polynomial $F(x_1, ..., x_n) = F(\underline{x}) = D(\underline{x}) - T(\underline{x})$, where the *diagonal* part $D(\underline{x})$ is a sum of terms of the form $b_i x_i^{2d}$ with all $b_i \geq 0$ and the *tail* $T(\underline{x})$ a sum of terms $a_{i_1 i_2 \ldots i_n} x_1^{i_1} \ldots x_n^{i_n}$ with $a_{i_1 i_2 \ldots i_n} > 0$ and at least two $i_\nu \geq 1$. We show that an arbitrary change of the signs of the tail terms of a positive semidefinite diagonal minus tail form will result in a sum of squares (sos) of polynomials. The work uses Reznick's theory of agiforms [3] and gives easily tested sufficient conditions for a form to be sos; one of these is piecewise linear in the coefficients of a polynomial and reminiscent of Lassere's recent conditions [2] but proved in completely a different manner.

Keywords

Polynomials (in several variables), Positive semidefiniteness, Sums of squares.

References

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