# Approximation by matrix-valued linear positive operators

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**Abstract**

Let $A = [a_{mn}]$ $(m, n \in \mathbb{N})$ be an infinite summability matrix. Then, for a given sequence $x = (x_n)_{n\in\mathbb{N}}$, the sequence $Ax = ((Ax)_m)_{m\in\mathbb{N}} = \left(\sum_{n=1}^{\infty} a_{mn}x_n\right)_{m\in\mathbb{N}}$ is called the $A$-transform of $x$ provided the series converges for each $m \in \mathbb{N}$. We say that $A$ is regular if $\lim_m (Ax)_m = L$ whenever $\lim_n x_n = L$. Assume now that $A$ is a non-negative regular summability matrix. Then, [1] introduced the concept of $A$-statistical convergence as follows: a sequence $x = (x_n)_{n\in\mathbb{N}}$ is said to be $A$-statistically convergent to a number $L$ if, for every $\varepsilon > 0$, $\lim_m \sum_{n:\,|x_n - L|\geq\varepsilon} a_{mn} = 0$. This limit is denoted by $st_A - \lim_n x_n = L$. It is easy to check that every convergent sequence is also $A$-statistically convergent for each non-negative regular summability matrix $A$; however the converse is not always true. Also, [2] proved that if $A = [a_{mn}]$ is any non-negative regular summability matrix for which $\lim_m \max_n\{a_{mn}\} = 0$, then $A$-statistical convergence is stronger than convergence. In this note, using the $A$-statistical convergence method which is weaker than the ordinary one, we obtain a Korovkin-type theorem for the approximation of a function by means of matrix-valued linear positive operators. We show that our theorem is stronger than the result introduced by [3]. Furthermore, we compute the $A$-statistical rates of our approximation.

## Keywords

Regular matrix, $A$-statistical convergence, Matrix-valued operators.

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