The equivalence of Error Orthogonal Models and Models with Commutative Orthogonal Block Structure

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Abstract

Models with orthogonal block structure (OBS), introduced by [5] still play an important part in experimental designs, see *e.g.* [1], [2].

We are interested in two important classes of these models, namely Error Orthogonal models (EO) and models with commutative orthogonal block structure (COBS).

EO, introduced by [7], are models whose least square estimators are uniformly best linear unbiased estimators and having the family of variance-covariance matrices given by

$$\mathbf{V} = \left\{ \sum_{j=1}^{m} \gamma_j \mathbf{Q}_j \right\}.$$

On the other hand, COBS, introduced by [3] are models whose orthogonal projection matrix on the space spanned by the mean vector commutes with $\mathbf{Q}_1, \ldots, \mathbf{Q}_m$, which are known pairwise orthogonal orthogonal projection matrices of a principal basis of a commutative Jordan algebra of symmetric matrices.

We intend to present the equivalence of these models making use of the model structure in order to estimate variance components.

Keywords

COBS, Error orthogonal models, Commutative Jordan algebras of symmetric matrices.

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