Entropy and trace inequalities

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Abstract

The notion of *entropy* was introduced in thermodynamics by Clausius in 1865, and since then many generalizations have been proposed with applications in different areas, such as statistical mechanics, information theory, etc. In this talk some mathematical aspects of the concept and some of its extensions are surveyed. Namely, a purely mathematical extension of the entropy for selfadjoint operators on a space with an indefinite norm is proposed and some operator inequalities inspired in well known results for Hilbert spaces are presented.

Keywords

Entropy, Operator theory, Krein spaces, Thermodynamics inequality, Peierls-Bogoliubov inequality.

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