Covariance matrix estimator of the causal effect estimator in a semi-parametric model

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Abstract

Causal effect estimator is often asymptotically normally distributed under standard regularity conditions. In addition, variance of this estimator is often constructed based on the availability of a covariance matrix. The covariance matrix which is obtained through Sandwich formula is often known as the robust covariance matrix estimator or the empirical covariance matrix estimator. It has achieved increasing use with the growing popularity in the field of the causal inference activities. In this study, we investigate the Sandwich estimation procedure for estimating the covariance matrix of parameter estimates in a semi-parametric model. Specifically, we show that the Sandwich covariance matrix estimate of the effect of exposure X on outcome Y is often far more variable than the usual parametric variance estimate and its coverage probabilities can be abysmal. In the following semi-parametric model [1],

$$E(Y_x|Z) = \psi x + g(Z)$$

where $\psi = E(Y_1 - Y_0)$ the average causal effect of being exposed and $g(Z)$ is a unknown function of the covariates Z, we will obtain the causal effect estimator ($\hat{\psi}$) by solving the following estimating equation

$$\sum_{i=1}^{n} \{X_i - P(X_i|Z_i)\} \left[Y_i - E(Y_i|Z_i) - \psi \{X_i - P(X_i|Z_i)\}\right] = 0$$

for ψ. We then derive the covariance matrix of $\hat{\psi}$ through Sandwich formula. We should indeed take into account all of the parameters in forming the associated covariance matrix [2], [3]. Furthermore, the covariance matrix for $\hat{\psi}$ is formed by obtaining the necessary derivatives.

Keywords

Robust covariance estimator, Estimating equations, Counterfactual outcomes, Sandwich covariance matrix.

References

[1] Babanezhad, M., S. Vansteelandt, and E. Goetghebeur (2010). Comparison of causal effect estimators under exposure misclassification. *J.* Statist. Plann. Inference 140, 1306-1319.

[2] Carroll, R.J. and G. Kauermann (2001). A note on the efficiency of sandwich covariance matrix estimation. *J.* Amer. Statist. Assoc. 96, 1387-1396.

[3] Zeileis, A. (2006). Object-oriented computation of sandwich estimators. *J.* Statist. 19, 1–16.