Some inequalities for unitarily invariant norms

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Abstract

We shall prove the inequalities

$$|||(A+B)(A+B)^*||| \leq |||AA^*+BB^*+2AB^*||| \leq |||(A-B)(A-B)^*+4AB^*|||$$

for all $n \times n$ complex matrices A, B and all unitarily invariant norms $|||\cdot|||$. If furhter A, B are Hermitian positive definite it is proved that

$$\prod_{j=1}^{k} \lambda_j(A\sharp_\alpha B) \leq \prod_{j=1}^{k} \lambda_j(A^{1-\alpha}B^\alpha), \quad 1 \leq k \leq n,\ 0 \leq \alpha \leq 1,$$

where $\sharp_\alpha$ denotes the operator means considered by [4] and $\lambda_j(X)$, $1 \leq j \leq n$, denote the eigenvalues of X arranged in the decreasing order whenever these all are real. A number of inequalities are obtained as applications.

Keywords

Singular values, Positive definite matrix, Unitarily invariant norms.

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