Generalizations of numerical range of matrix polynomials

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Abstract

Let $L(\lambda) = \sum_{i=0}^{m} \lambda^i A_i$ be a matrix polynomial, where the coefficients A_i are complex $n \times n$ matrices. For integer $k \geq 1$, the set

$$\Lambda_k(L(\lambda)) = \\ = \{\lambda : PL(\lambda)P = 0_n \text{ for some } k - \text{rank orthogonal projection } P\}$$

is defined as *higher rank numerical range* of $L(\lambda)$ [1]. For $k = 1$, $\Lambda_1(L(\lambda))$ reduces to the numerical range of a matrix polynomial.

In this study, we are interested in essential algebraic and geometric properties of the set. We present necessary and sufficient conditions for the set to be bounded and we investigate the number of its maximal connected subsets. The boundary of $\Lambda_k(L(\lambda))$ is also examined concerning its corner points. Further, our survey [2] elaborates the number of elements in $\Lambda_k(L(\lambda))$ of maximum modulus, for special cases of matrix polynomials $L(\lambda)$ and their location on the complex plane, extending the results in [3], [4].

Keywords

Numerical ranges, Matrix polynomials.

References

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