Applications of Matrix Algebra in Various Fields

Ali S. Hadi

Department of Mathematics and Actuarial Science
The American University in Cairo, Egypt

Intended Audience and Purpose

These lectures are intended for graduate students and researchers and practitioners who are interested in the application of matrix algebra in their own fields. They are not designed for researchers in matrix theory/linear algebra. The purpose of these lectures is to acquaint graduate students, researchers, and practitioners with new as well as standard applications of matrices in statistics and graph theory. The lectures have one theme in common, that is, the applications of matrix algebra in various fields; other than that, they are unrelated to each other, that is, no one is a prerequisite for the other. Nothing is new in Lectures 1 and 2, but these are standard applications of matrix algebra that I personally found to be interesting. Lectures 3 and 4 are recent but modest contributions to matrix algebra that are of interest to people who use statistics in their work, particularly linear models and multivariate analysis techniques.

Pre-requisite background

Elementary matrix algebra including eigenvalue and some familiarity with quadratic forms and formula for ellipsoids.

Lecture 1. Matrices Useful for Graph Theory and Expert Systems

Graphs (where each variable is represented by a node and the relationships among them are represented by arcs or arrows connecting the nodes) are used for solving problems, for example, in the area of artificial intelligence. In this lecture we discuss how these graphs can be represented numerically using certain types of matrices such as the adjacency and attainability matrices. From this numerical representation several characteristics of the graphs can be easily obtained.

Convenient reference
Castillo, E., Gutiérrez, J. M., Hadi, A. S. (1997). *Expert Systems and Probabilistic Network Models*. New York: Springer-Verlag.

Lecture 2. Interesting Applications of Eigenvalues and Eigenvectors

Eigenvalues and vectors have numerous and interesting applications in many fields. These applications are seldom discussed in linear/matrix algebra courses. In this lecture I will provide some applications of the eigenvalues and vectors that I personally found to be interesting (for example, quadratic forms, minimum volume ellipsoids or ellipsoidal hull).

Convenient reference

Hadi, A. S. (1996). *Matrix Algebra as a Tool.* Belmont, CA: Duxbury Press.

Lecture 3. Regularization of Ill-Posed Systems

Regularization methods are used to solve ill-posed problems which appear frequently in science, engineering and statistics. Solving these problems is difficult when the system is ill-posed or ill-conditioned. Regularization methods, such as Tikhonov regularization (TR), are well known for solving ill-posed systems. The accuracy of the TR solution depends on a regularization factor α, an optimal value of which is hard to find. In fact, the optimal value often does not exist. We present a new regularization method that does not require finding the optimal regularization factor. The theoretical framework of the proposed method is presented and is shown not to depend on the optimal value of α. The accuracy of the proposed method is illustrated using several data sets. The method is compared to that of the TR method and is shown to outperform the TR method.

Convenient reference

Moustafa, R. E., Hadi, A. S. (2010). A Method for Regularization of Ill-Posed Systems. Proceedings of *The First International Conference on Mathematics and Statistics*, American University of Sharja.

Lecture 4. Preprocessing the Rows of Multivariate Data Grossly Distorts Their Graphical and Correlation Structures

Before performing certain statistical analysis methods (*e.g.*, principal components and factor analyses), it may be necessary to preprocess the data to make them suitable for the analysis. For example, given an $n \times p$ data matrix X, which represents n multivariate observations on p variables, the columns and/or the rows of X may be centered and/or scaled before applying a statistical method to the data matrix X. In this lecture we show with numerical examples as well as mathematical proofs that centering and/or scaling the rows of X distorts the graphical structure of the observations in the multi-dimensional space and substantially alters the correlation structure among the variables. Accordingly, analysts who use such row centering and/or scaling should first demonstrate that the process results in a new, more appropriate structure for their questions.